The 19th North American Meeting of the Urban Economics Association
@ Université du Québec à Montréal, Montréal, Canada

Understanding Regional Dynamics

October 3, 2025

Minoru OSAWA

Institute of Economic Research, Kyoto University, Japan

Broad motivation

- Quantitative spatial models typically rely on equilibrium uniqueness to conduct unambiguous conterfactual analyses
- By design, QSMs do "not aim to provide a fundamental explanation for the agglomeration of economic activity" (Redding & Rossi-Hansberg, 2017)
- Agglomeration in these models are due to differences in "unobserved fundamentals" or "first nature" of Krugman (1993)
- Under big shocks and/or alternative possibilities, agglomeration forces and multiple equilibria can be important (Bleakley & Lin, 2012; Lin & Rauch, 2022)
- How **models of spatial agglomeration** behaves in this case?
  What **spatial patterns** may be explained/represented by "second nature"?

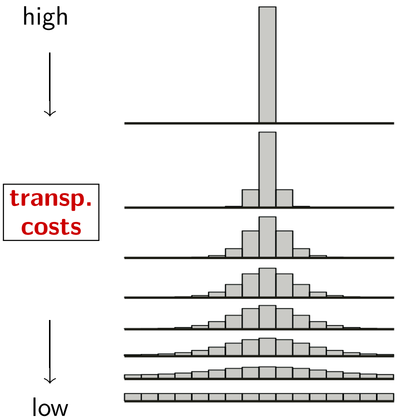
Static regional models and their taxonomy (Akamatsu *et al.*, 2017)

- Two general types of dispersion forces
  - Crowding *within* each location : "**local**" dispersion forces
    e.g., inelastic supply of housing (nontraded good) (Helpman, 1998)
  - Crowding *across* locations : "**global**" dispersion forces
    e.g., immobile factor + trade (Krugman, 1991)
- Implied model types:

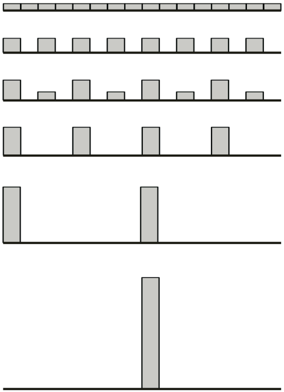
	Local	Global	Notable instances
1. Type L	✓		Helpman (1998); Redding & Sturm (2008); Allen & Arkolakis (2014)
2. Type G		✓	Krugman (1991); Puga (1999); Forslid & Ottaviano (2003)
3. Type LG	✓	✓	Tabuchi (1998); Pflüger & Tabuchi (2010); Kucheryavyy *et al.* (2024)

- Notably, the great majority of conventional QSMs are **Type L** (Redding, 2025)

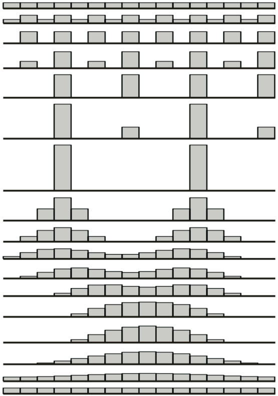
Static models: Different type, different spatial implications



(A) Type L



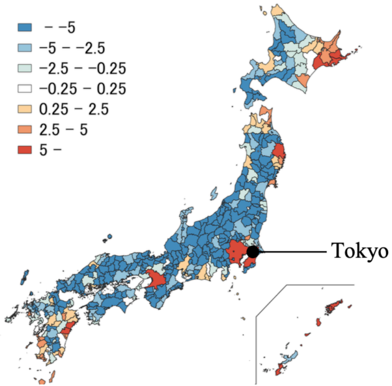
(B) Type G



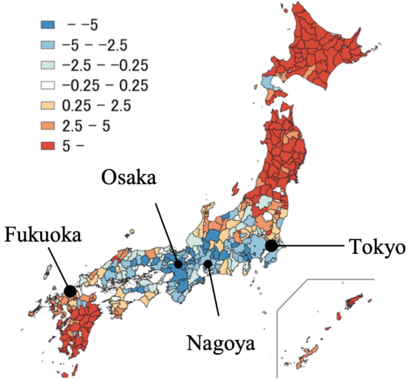
(C) Type LG

Static models: Different type, different spatial implications

- Spatial implications are in the *opposite* directions (Sugimoto *et al.*, 2025)



(B) Removal of highways (Type L)



(C) Removal of highways (Type LG)

This study

- Extend this research program to an explicitly dynamic setting.
- As a specific example, we examine **Allen & Donaldson (2020)** [**AD**]

  "Persistence and Path Dependence in the Spatial Economy" NBER w28059

  - A good starting point: Clean, tractable, & various microfoundations
  - Can be seen as a dynamic version of Allen & Arkolakis (2014)
    $\Rightarrow$ Should resemble "Type L" static models $\cdots$ We will confirm this.
- Approach: Agglomeration as instability of symmetry (Papageorgiou & Smith, 1983)
  - e.g., New Economic Geography

Q1. How *endogenous forces* drive agglomeration?

Q2. What *spatial patterns* can emerge?

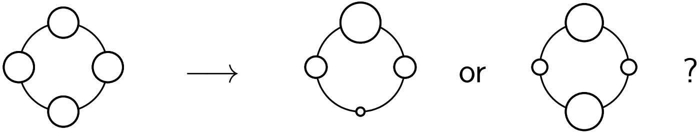
The (symmetric) AD framework

We shall stick to the most symmetric version.

- N locations with homogeneous characteristics
- Iceberg trade frictions btw. locations $\{\tau_{ij}\}$, $\tau_{ij} \geq 1$
- Iceberg migration frictions btw. locations $\{\mu_{ij}\}$, $\mu_{ij} \geq 1$
- Population distribution $\boldsymbol{L}_t = (L_{i,t})_{i=1}^N$ at time $t \in \{0, 1, 2, \cdots\}$
- Perfectly competitive Armington with *local* but intertemporal externalities.
  - Amenity at time t: $u_i(\boldsymbol{L}) = L_{i,t}^{\beta_1} \cdot L_{i,t-1}^{\beta_2}$ ($\beta_1 < 0$, $\beta_2 > 0$)
  - Productivity at time t: $a_i(\boldsymbol{L}) = L_{i,t}^{\alpha_1} \cdot L_{i,t-1}^{\alpha_2}$ ($\alpha_1 > 0$, $\alpha_2 > 0$)
- Market/migration eqm. defines discrete-time dynamics: $\boldsymbol{L}_t = \boldsymbol{F}(\boldsymbol{L}_{t-1})$.

Symmetric four-location economy: A minimal testbed

- By assuming a symmetric geographical setting, we can focus on the *symmetric steady-state equilibrium* $\bar{\boldsymbol{L}} = (\frac{1}{N}, \frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ because $\bar{\boldsymbol{L}} = \boldsymbol{F}(\bar{\boldsymbol{L}})$.
- Instability of $\bar{\boldsymbol{L}}$ $\Rightarrow$ Some form of "endogenous" agglomeration.
- The four-location circular economy makes analysis simple yet relevant:



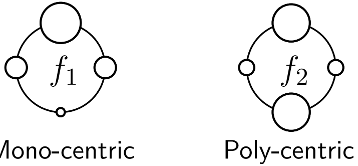
- In fact,
  - Type L static models: Only a single-peaked agglomeration.
  - Type G static models: Poly-centric agglomeration (multiple cities).

Stability of symmetric steady state

- Friction matrices have special structures:

$$\begin{bmatrix} 1 & \phi & \phi^2 & \phi \\ \phi & 1 & \phi & \phi^2 \\ \phi^2 & \phi & 1 & \phi \\ \phi & \phi^2 & \phi & 1 \end{bmatrix} \qquad \phi = \begin{cases} r \in (0,1) & \text{(freeness of trade)} \\ s \in (0,1) & \text{(freeness of migration)} \end{cases}$$

- This allows for the analytical characterization of stability of $\bar{\boldsymbol{L}}$.
- If the absolute value of the "**net agglomeration forces**" f_1 and f_2 ($\approx$ agglom. force $\div$ disp. force) are smaller than 1, $\bar{\boldsymbol{L}}$ is stable.



Net agglomeration forces in the AD framework

Concretely, for $k = 1, 2$,

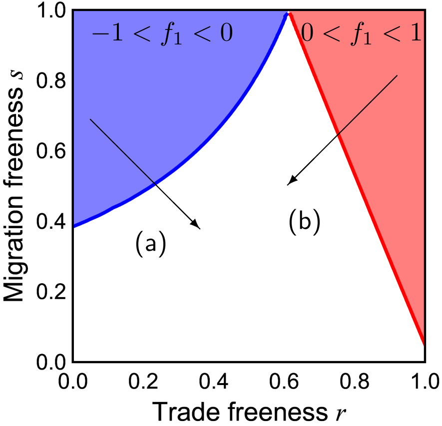
$$f_k = \frac{f_k^\sharp}{f_k^\flat} \quad \text{where} \quad \begin{cases} f_k^\sharp = \alpha_2 A_k + \beta_2 + \lambda_k \theta^{-1}(1-\lambda_k^2)^{-1}, \\ f_k^\flat = -\alpha_1 A_k - \beta_1 + B_k + \theta^{-1}(1-\lambda_k^2)^{-1}, \end{cases}$$

$$A_k = \frac{\chi_k + (\sigma - 1)(1+\chi_k)}{1 + (\sigma - 1)(1+\chi_k)} \in (0,1), \quad B_k = \frac{1-\chi_k}{1+(\sigma-1)(1+\chi_k)} > 0.$$

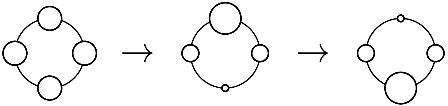
- σ: Armington CES elasticity.
- θ: Migration friction (Fréchet dispersion parameter)
- $\chi_k \in (0,1)$: a trade cost index, $\lambda_k \in (0,1)$: a migration cost index.

Agglomeration as instability of symmetry (1/3)

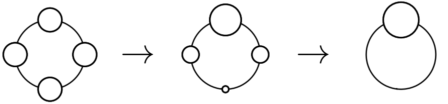
Stability region for the monocentric direction ($|f_1| < 1$)



(a) Agglomeration with oscillation.



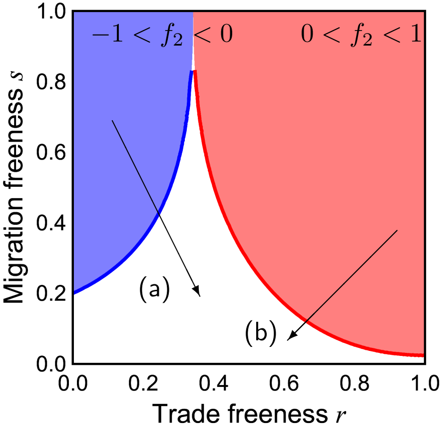
(b) Agglomeration without oscillation.



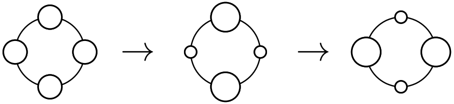
$(\sigma, \theta, \alpha_1, \beta_1, \alpha_2, \beta_2) = (8, 6, 0.7, -0.4, 0, 0)$.

Agglomeration as instability of symmetry (2/3)

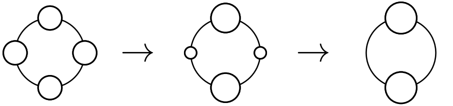
Stability region for the polycentric direction ($|f_2| < 1$)



(a) Agglomeration with oscillation.



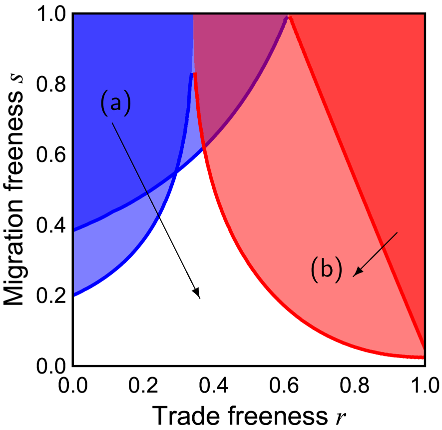
(b) Agglomeration without oscillation.



$(\sigma, \theta, \alpha_1, \beta_1, \alpha_2, \beta_2) = (8, 6, 0.7, -0.4, 0, 0).$

Agglomeration as instability of symmetry (3/3)

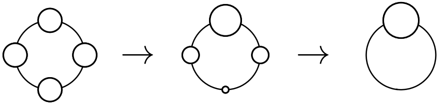
The stability region of symmetry $\bar{\boldsymbol{L}}$: $|f_1| < 1$ *and* $|f_2| < 1$



(a) Agglomeration with oscillation.

- Both mono-centric & poly-centric spatial patterns

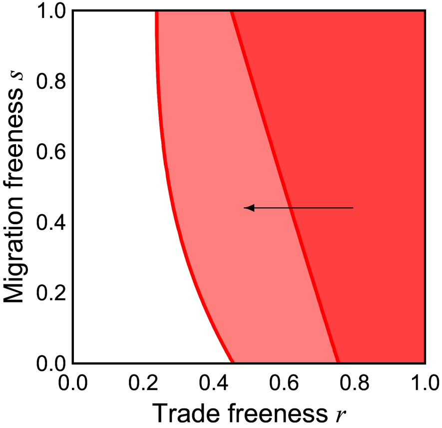
(b) Agglomeration without oscillation.



- Only mono-centric spatial patterns

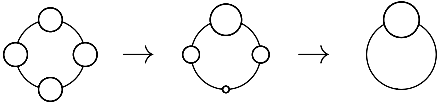
$(\sigma, \theta, \alpha_1, \beta_1, \alpha_2, \beta_2) = (8, 6, 0.7, -0.4, \mathbf{0}, \mathbf{0})$.

A more relevant case: Estimates for the US from AD



$(\sigma, \theta, \alpha_1, \beta_1, \alpha_2, \beta_2) = (5, 6, 0.3, -0.4, 0.1, 0.3)$

- Agglomeration without oscillation.



- Only monocentric spatial patterns.
- Similar to the Allen–Arkolakis model in the static world.

Summary

- "Endogenous" spatial patterns in the Allen–Donaldson framework
- As expected, behavior similar to the Allen–Arkolakis model ($\approx$ "Type L")

- Simple geographical settings are still important in understanding the basic mechanics of spatial models with both agglomeration and dispersion forces.
- In doing so, having four locations is crucial for studying spatial patterns.
- What can be said for the empty cells? Also, quantitative relevance?

	Type L	Type G	Type LG
Static	Helpman (1998), Allen & Arkolakis (2014)	Krugman (1991), Puga (1999) (§3)	Tabuchi (1998), Kucheryavyy *et al.* (2024)
Dynamic	Allen & Donaldson (2020)	**???**	**???**

References

Akamatsu, Mori, Osawa, & Takayama (2017). Spatial scale of agglomeration and dispersion: Number, spacing, and the spatial extent of cities. Unpublished manuscript (Updated 2025).

Allen & Arkolakis (2014). Trade and the topography of the spatial economy. *The Quarterly Journal of Economics*.

Allen & Donaldson (2020). Persistence and path dependence in the spatial economy. NBER w28059.

Bleakley & Lin (2012). Portage and path dependence. *The Quarterly Journal of Economics*, **127**.

Forslid & Ottaviano (2003). An analytically solvable core-periphery model. *Journal of Economic Geography*, **3**.

Helpman (1998). The size of regions. In Pines, Sadka, & Zilcha, editors, *Topics in Public Economics*, pages 33–54. Cambridge University Press Cambridge.

Krugman (1991). Increasing returns and economic geography. *Journal of Political Economy*, **99**.

Krugman (1993). First nature, second nature, and metropolitan location. *Journal of Regional Science*, **33**.

Kucheryavyy, Lyn, & Rodríguez-Clare (2024). Spatial equilibria: The case of two regions. *Journal of International Economics*, **152**.

Lin & Rauch (2022). What future for history dependence in spatial economics? *Regional Science and Urban Economics*, **94**.

Papageorgiou & Smith (1983). Agglomeration as local instability of spatially uniform steady-states. *Econometrica*, **51**.

Pflüger & Tabuchi (2010). The size of regions with land use for production. *Regional Science and Urban Economics*, **40**.

Puga (1999). The rise and fall of regional inequalities. *European economic review*, **43**.

Redding (2025). Spatial economics.

Redding & Rossi-Hansberg (2017). Quantitative spatial economics. *Annual Review of Economics*, **9**.

Redding & Sturm (2008). The costs of remoteness: Evidence from german division and reunification. *American Economic Review*, **98**.

Sugimoto, Takayama, & Takagi (2025). A quantitative spatial model for evaluating transport-induced spatial reorganization. *Transport Policy*, **172**.

Tabuchi (1998). Urban agglomeration and dispersion: A synthesis of alonso and krugman. *Journal of Urban Economics*, **44**.